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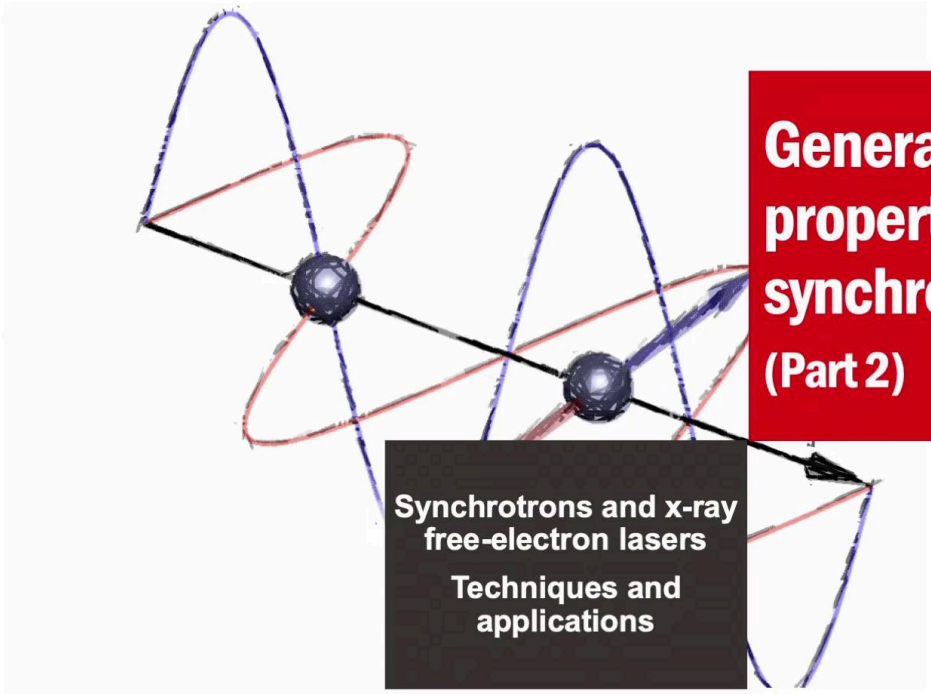


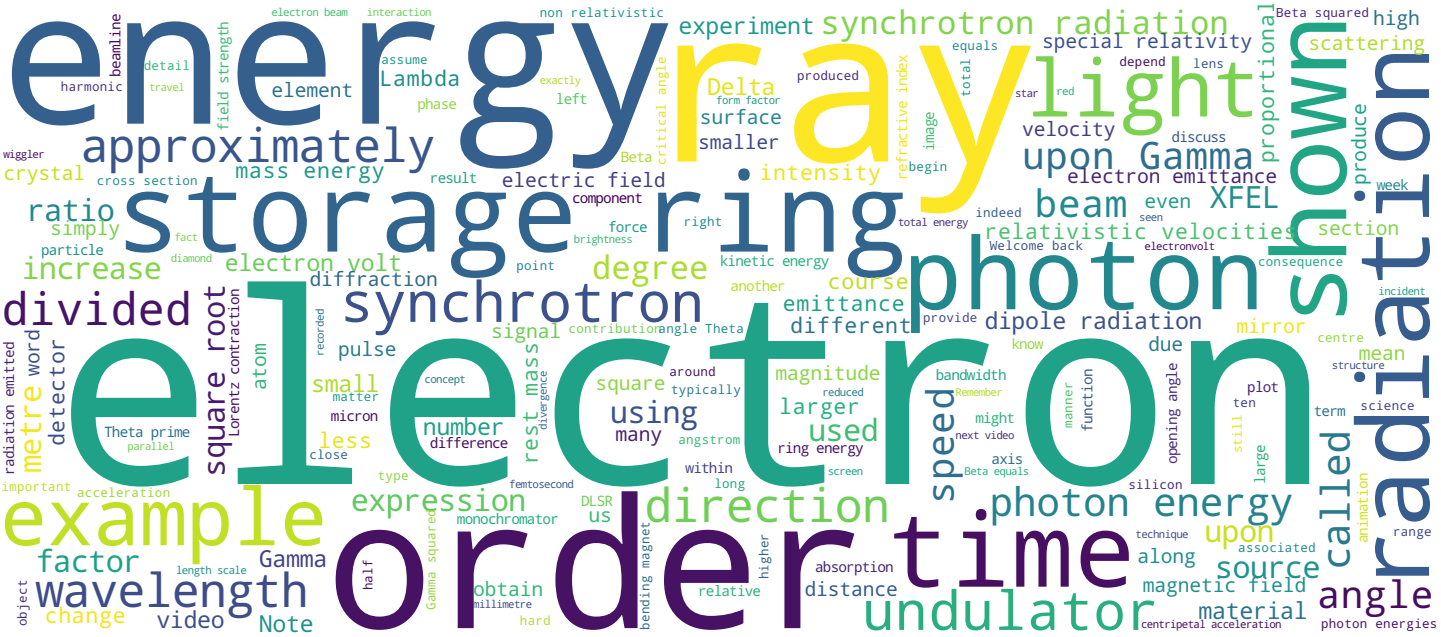
# Generation and properties of synchrotron light (Part 2)

## Synchrotrons and x-ray free-electron lasers

## Techniques and applications

École polytechnique fédérale de Lausanne

Prof. Philip Willmott

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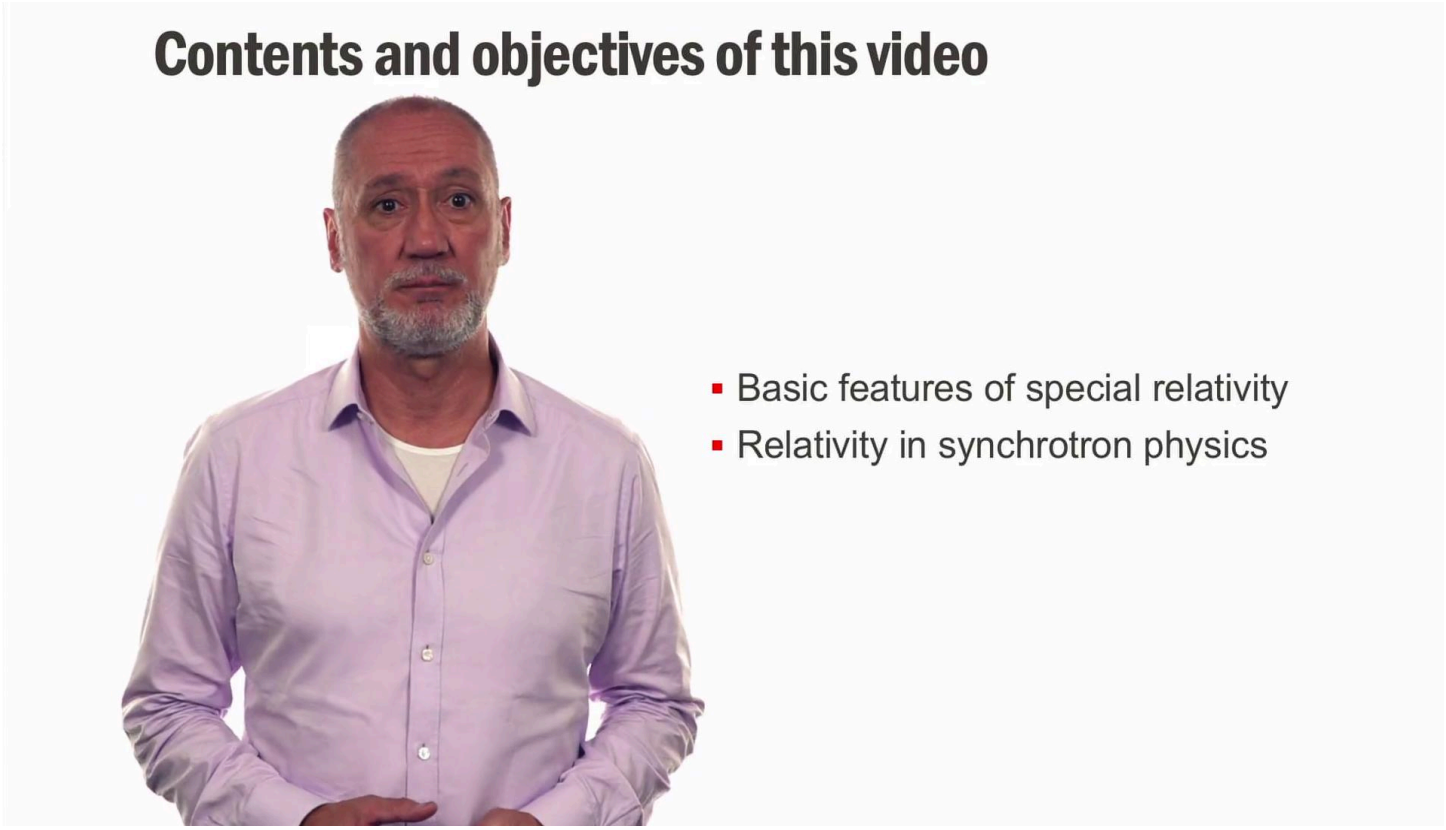



Welcome back. In this video, we will discover that because the electrons in storage rings have velocities very close to the speed of light, the radiation they produce is very different from radiation emitted by accelerated electrons at non relativistic velocities. This will prepare us for seeing why synchrotron radiation is so collimated and intense.

Notes

Summary

0m 05s

# Why the Albert Einstein are we learning relativity?

- Electrons in storage ring close to speed of light
  - e.g. c – v @ ESRF = 1.1 m $s^{-1}$
- Mass of these electrons in lab frame ca. 2000 – 20000 times its rest mass
  - Determines magnitude of forces needed to maintain electrons on closed path
- Light emission from relativistic electrons highly anisotropic cf. nonrelativistic electrons
- Optical power output from relativistic electrons in forward direction enormously increased

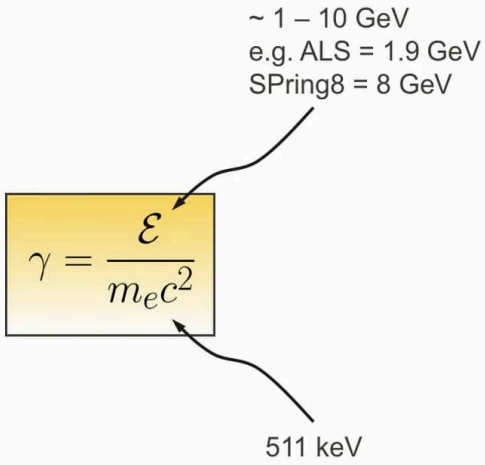
So why do we need to delve into special relativity? If one were to ignore special relativity and try to calculate the velocity of, say, a three giga electron volt electron using simply Newtonian mechanics, one would arrive at a speed of 2.3 x 10 to the 10 metres per second, nearly 80x the speed of light. But of course, we know we can't ignore special relativity. It emerges that electrons in storage rings have velocities only a couple of metres per second, less than the ultimate speed limit of nature, the speed of light. For the example of the ESRF, the difference is equivalent to a relaxed stroll. Special relativity states that as an object approaches the speed of light. Its mass increases in such a manner that it can never actually reach that ultimate velocity. As we will shortly see, the mass of an electron in storage rings, as observed by us in the laboratory frame, is several thousand times greater than if the electron were at rest, relative to us. This is important as this increased mass will determine the magnitude of the forces that must be applied to maintain these high energy electrons on a closed path within the storage ring. Moreover, space becomes distorted at relativistic velocities, which will have a crucial impact on the shape and also the intensity of the radiation emitted by the electrons as they are forced to execute curved paths.

Notes

Summary

0m 30s

# The Lorentz factor γ




$\gamma \sim 2000 \rightarrow 20\,000$

- Dimensionless
- Used in many expressions in synchrotron physics
  - Opening angle of synchrotron radiation
  - Relativistic mass of electrons
  - Relativistic Doppler shift
  - Lorentz contraction
  - Radiative power from synchrotrons
  - Characteristic photon energy of synchrotron light
  - …

We now introduce the dimensionless parameter known as the Lorentz factor, denoted by the Greek symbol Gamma. This is simply the ratio of the electron storage ring energy to their rest mass energy, this being 511 KeV. Gamma therefore assumes values of the order of a few thousand for typical storage ring energies. Gamma crops up again and again in expressions in synchrotron physics, most notably in the opening angle of synchrotron radiation, the relativistic mass of electrons, the relativistic Doppler shift and Lorentz contraction, the radiative power produced by synchrotrons and the characteristic photon energy, produced by synchrotrons. To name, just a few examples.

Notes

Summary

2m 08s

# Relativistic energy and mass

- Total relativistic energy of particle of mass $m_e$ travelling at velocity v

$$\mathcal{E} = \frac{m_e c^2}{\sqrt{1-(v/c)^2}} = \frac{m_e c^2}{\sqrt{1-\beta^2}}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- This is equal to the rest mass energy + kinetic energy

$$\mathcal{E} = m_e c^2 + \mathrm{KE}$$

- Therefore

$$\mathrm{KE} = m_e c^2 \left( \frac{1}{\sqrt{1-\beta^2}} - 1 \right)$$

- $v \ll c$

$$\left(1-\beta^2\right)^{-1/2} \approx 1 + \beta^2/2 \Rightarrow$$

$$\mathrm{KE} \approx \frac{m_e v^2}{2}$$

- $(c - v) \ll c$

$$\mathrm{KE} \approx \mathcal{E} = \frac{m_e c^2}{\sqrt{1-\beta^2}} = \gamma m_e c^2$$

- $v = c$

$$\mathrm{KE} = \infty$$

Einstein derived an equation for the total energy of a particle as being equal to the rest mass energy divided by the square root of 1 minus V/C squared. The ratio V/P is also denoted by Beta, but this should not be confused with the Beta in last week's videos describing the absorption index. But we know that Gamma is the ratio of the particle energy to the rest mass energy. Hence, we can immediately say that Gamma is equal to 1 upon the square root of 1 minus Beta squared. The kinetic energy of a particle is therefore equal to the total energy minus the rest mass energy, which is therefore equal to MeC squared multiplied by 1 over the square root of 1 minus Beta squared minus 1. Let us consider three different cases for this expression. First, for small velocities far below C, the first term 1 upon the square root of 1 minus Beta squared can be accurately approximated as 1 plus Beta squared upon 2, which directly leads to the kinetic energy being equal to MeV squared upon 2 as predicted by Newtonian mechanics. For velocities close to but below C, the kinetic energy is approximately equal to Gamma MeC squared. In other words, the rest mass energy contributes to the total energy only insignificantly.

Notes

Summary

2m 57s

# β and γ

- We know

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- Therefore

$$\beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2}$$

- But

$$\frac{1}{\gamma^2} \ll 1$$

- Therefore

$$\beta \approx \left(1 - \frac{1}{2\gamma^2}\right)$$

N.B. $(1+x)^n \approx 1 + nx; \quad x \ll 1$

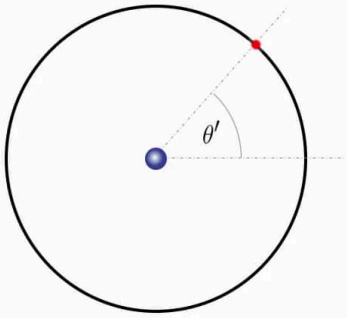
Lastly, at V exactly equal to C the mass and energy are infinite, no particle with mass can assume this value. Only the massless particles, photons, gluons and probably gravitons move at the speed of light. We know how to express Gamma in terms of Beta, but we'd also like to do the reverse. From our expression for Gamma, we can square both sides and obtain the expression Beta equals 1 minus 1 upon Gamma squared to the half. But for relativistic particles, 1 upon Gamma squared is much smaller than unity, so we can use the Taylor expansion of 1 plus X to the N, for small x to obtain Beta equals 1 minus 1 upon 2 Gamma squared.

Notes

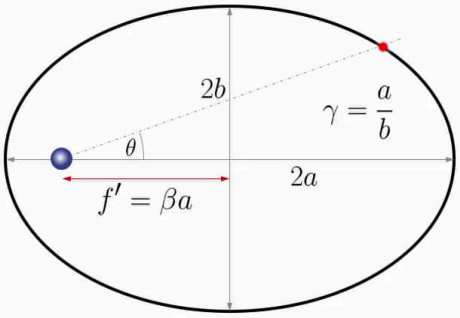
Summary

4m 31s

# Space distortion by special relativity




$v = 0$




$v > 0$

$$\beta = v/c$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\theta = \sin^{-1}\left[\frac{\sin\theta'}{\gamma(1+\beta\cos\theta')}\right]$$

See also “Mr Tompkins in Paperback”, George Gamow (Canto Classics)

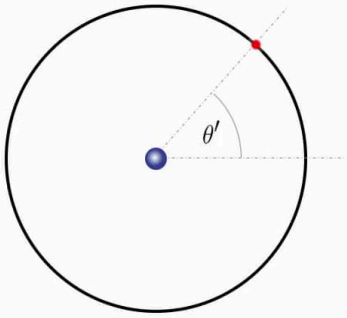
Imagine now you are in a spaceship moving at modest, non relativistic speeds. Incidentally, the fastest ever manmade macroscopic object was Helios A, a solar probe that achieved a velocity of 70.220 metres per second, less than a quarter of a 1000th of the speed of light, and only about 12% of the speed of a one electron volt electron. Anyway, back to our spaceship from the billions of stars in the firmament. You make a selection of those with the same brightness and the same distance from your location in space. Now, one of the crew members accidentally hooks his walking stick around the accelerator pedal and woosh, off you go at relativistic velocities, the stars that seemed before to be equal intensity and equally distant from you rearrange themselves so that space seems to have been wrapped forward. The stars behind you become redder and dimmer while those in front become brighter and bluer. All this is a consequence of special relativity. It lies well outside the remit of this course to provide all the background, physics and mathematics of special relativity, well outside. For a gentle introduction to the curious student with no prior knowledge of this subject, however, I strongly recommend Mr.

Notes

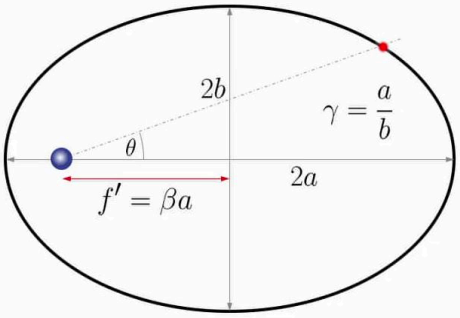
Summary

5m 23s

# Space distortion by special relativity




$v = 0$




$v > 0$

$$\beta = v/c$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\theta = \sin^{-1}\left[\frac{\sin\theta'}{\gamma(1+\beta\cos\theta')}\right]$$

See also “Mr Tompkins in Paperback”, George Gamow (Canto Classics)

Tompkins in paperback by George Gamow. We limit ourselves here to a geometrical description of how space becomes distorted at relativistic velocities. Consider a celestial spherical surface, the centre of which you sit when at rest relative to that surface. You observe a star sitting on the sphere at an angle Theta prime. If you travel at a velocity V relative to the sphere, this becomes ellipsoidal with a ratio of the semi major to semi minor axes, A upon B equal to Gamma, you now sit at the left [inaudible 00:07:38] away from the centre of the ellipsoid. By a distance Beta times A. Remeasuring the location of the same star, it will now appear to be at a new angle Theta given by the expression shown here. Let's consider a modest case of Beta equals 0.75. Gamma is then 4, divided by the square root of 5, which is equal to 1.789 equal to the ratio of A upon B. At zero velocity, you recorded a star exactly to your left. That is at Theta prime equals 90 degrees. At your new velocity, it now appears to be at an angle of only 34 degrees to the direction of travel. And this is for a Gamma of only 1.789 Things become far, far, far more squished into the forward direction at the kind of velocities associated with storage ring electrons.

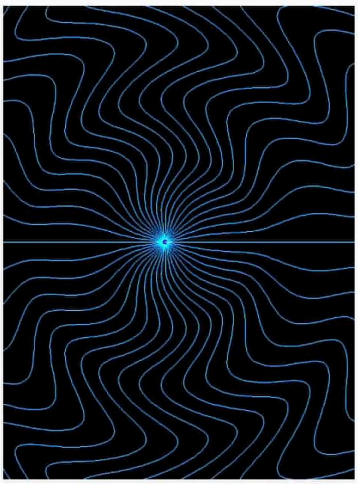
Notes

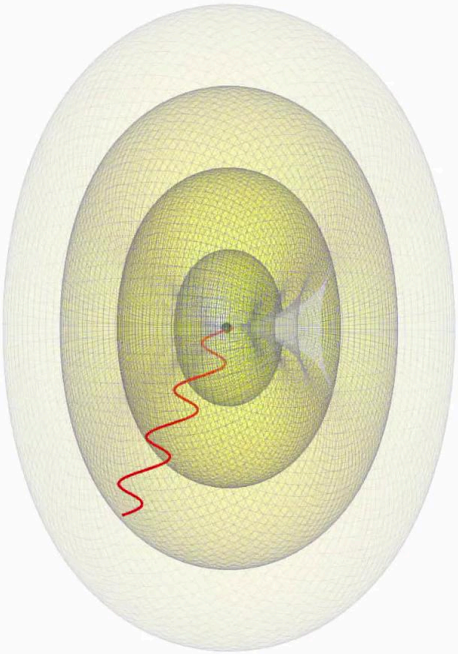
Summary

6m 57s

# From dipole to synchrotron radiation

Remember dipole radiation from last week:





We now return to dipole radiation shown again on the left as a 2D animation of the electric field lines and on the right as a 3-D cartoon of the expanding surfaces on the same phase. The election is being accelerated to and fro in the horizontal plane in both cases.

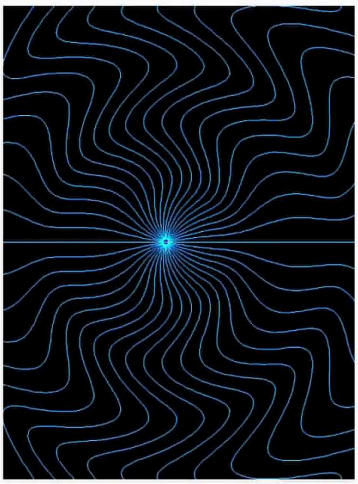
Notes

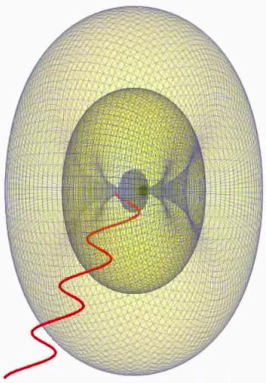
Summary

8m 40s

# From dipole to synchrotron radiation

Remember dipole radiation from last week:





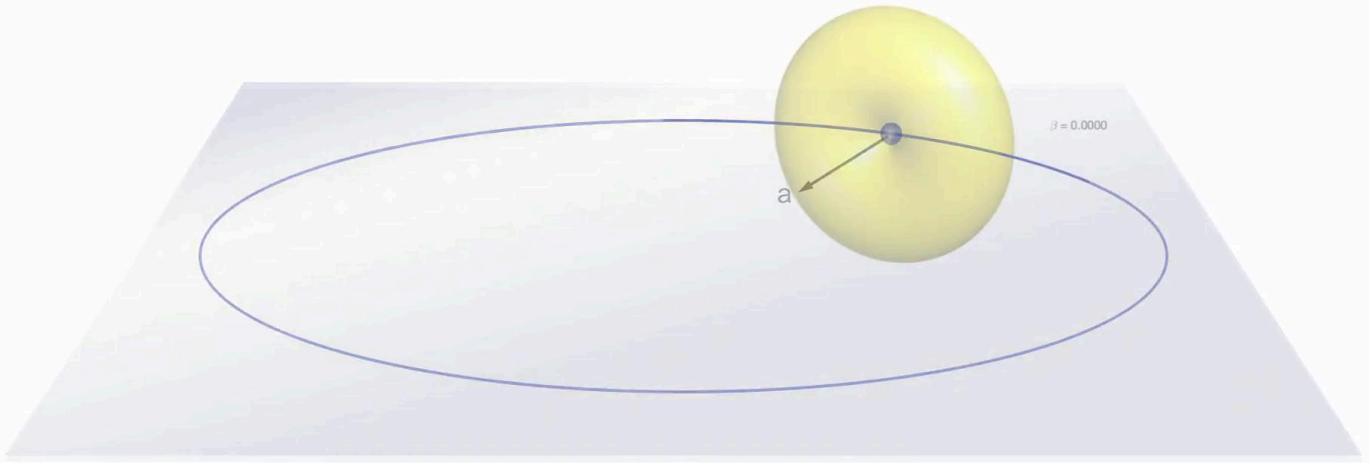
Remember also that along the axis of acceleration, dipole radiation shows a node of zero amplitude. Keep these images in mind.

Notes

Summary

9m 04s

# From dipole radiation to synchrotron radiation



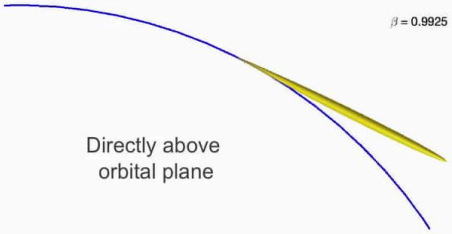

Consider now an electron in a circular orbit moving at non relativistic velocities. The centripetal acceleration A is towards the centre of the orbit, we therefore expect the dipole radiation to have its node also along the direction of acceleration.

Notes

Summary

9m 14s

# From dipole radiation to synchrotron radiation




If we now increase the electron speed, the dipole radiation it emits will distort in the manner we have just described for our spaceship and stars example. If we change our perspective to being directly above the playing of the electron orbit, we see still more clearly how the radiation transforms from having a doughnut shape at low velocities to being extremely forward, directed as the velocity approaches the speed of light. In this animation, I have progressively scaled back the intensity as the electron reaches higher and higher speeds in order to keep the general shape of the emerging synchrotron radiation within the screen. Had I not done this before, the forward directed intensity would have shot way off the bottom right hand end of the screen and I mean way off. We discuss the power dependence on Gamma quantitatively in just a moment.

Notes

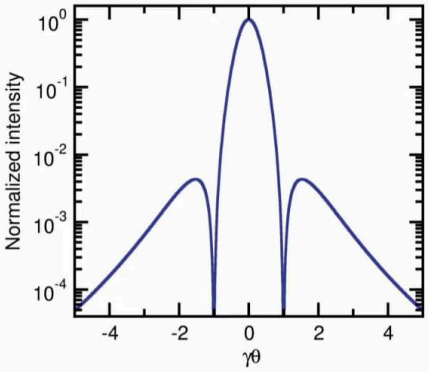
Summary

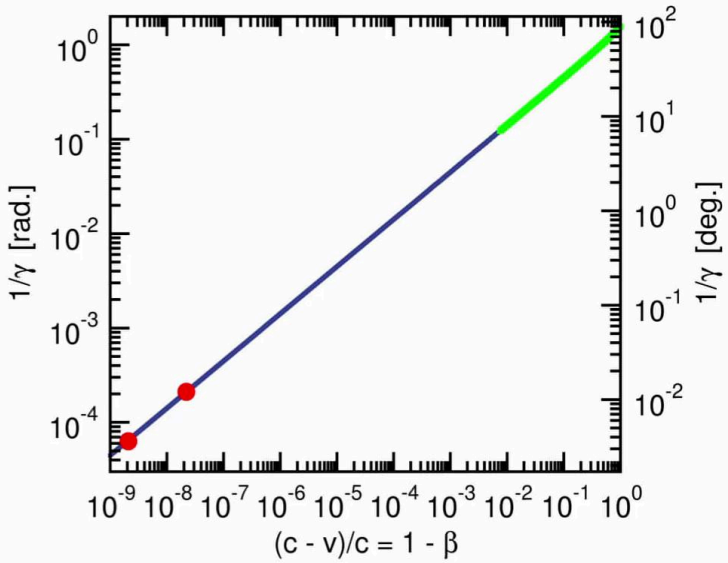
9m 35s

# From dipole to synchrotron radiation

$$\theta = \sin^{-1}\left[\frac{\sin\theta'}{\gamma(1+\beta\cos\theta')}\right]$$

θ' in frame of reference of electron = π/2
θ in observer's F.o.R. = 1/γ
Opening angle = ± 1/γ
FWHM ≃ 1/γ




Remember that at the beginning of the animation we have just seen, we start with dipole radiation with the nodal axis pointing in the direction of the centripetal acceleration, but the axis of this node sweeps forward as the electron speeds up. The angle it assumes for a given Gamma is easy to calculate using our angle transformation equation shown again here. In the stationary frame, Theta prime is equal to 90 degrees and sign Theta prime is equal to 1 and cos Theta prime is equal to zero. Hence Theta is equal to the inverse sign of 1 upon Gamma, which for large Gamma values is simply equal to 1 upon Gamma as long as Gamma is expressed in radians. Hence, the forward pointing cone of radiation lies entirely within plus or minus 1 upon Gamma and the full width half maximum is approximately equal to 1 upon Gamma. As Gamma might be of the order of 5000, it is immediately clear that synchrotron radiation is extremely collimated, with opening angles of a fraction of a Millirad or about 0.01 degrees, 0.01 of a degree. A plot of the normalised intensity as a function of Theta times Gamma is shown bottom left on the right is a plot of the synchrotron radiation opening angle once upon Gamma as a function of 1 minus V upon C.

Notes

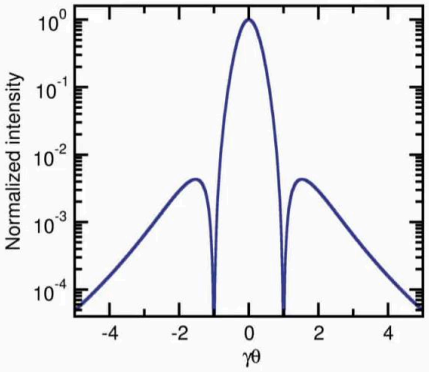
Summary

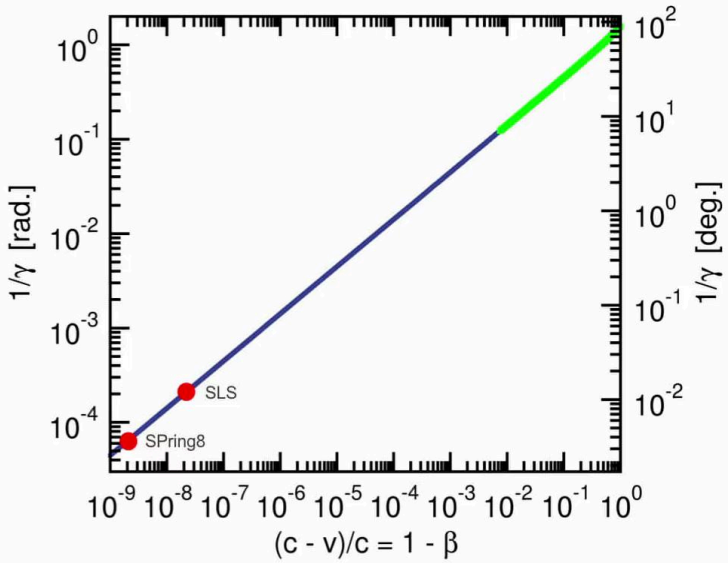
10m 36s

# From dipole to synchrotron radiation

$$\theta = \sin^{-1}\left[\frac{\sin\theta'}{\gamma(1+\beta\cos\theta')}\right]$$

θ' in frame of reference of electron = π/2
θ in observer's F.o.R. = 1/γ
Opening angle = ± 1/γ
FWHM ≃ 1/γ



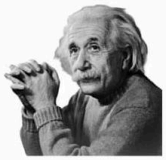

Note that the animation we just saw only covers the angular range highlighted in green, the natural opening angles of the SLS with the storage ring energy of 2.4 GeV and spring 8 with its energy of 8 GeV, are shown here.

Notes

Summary

12m 06s

# Doppler + Einstein = synchrotron x-rays

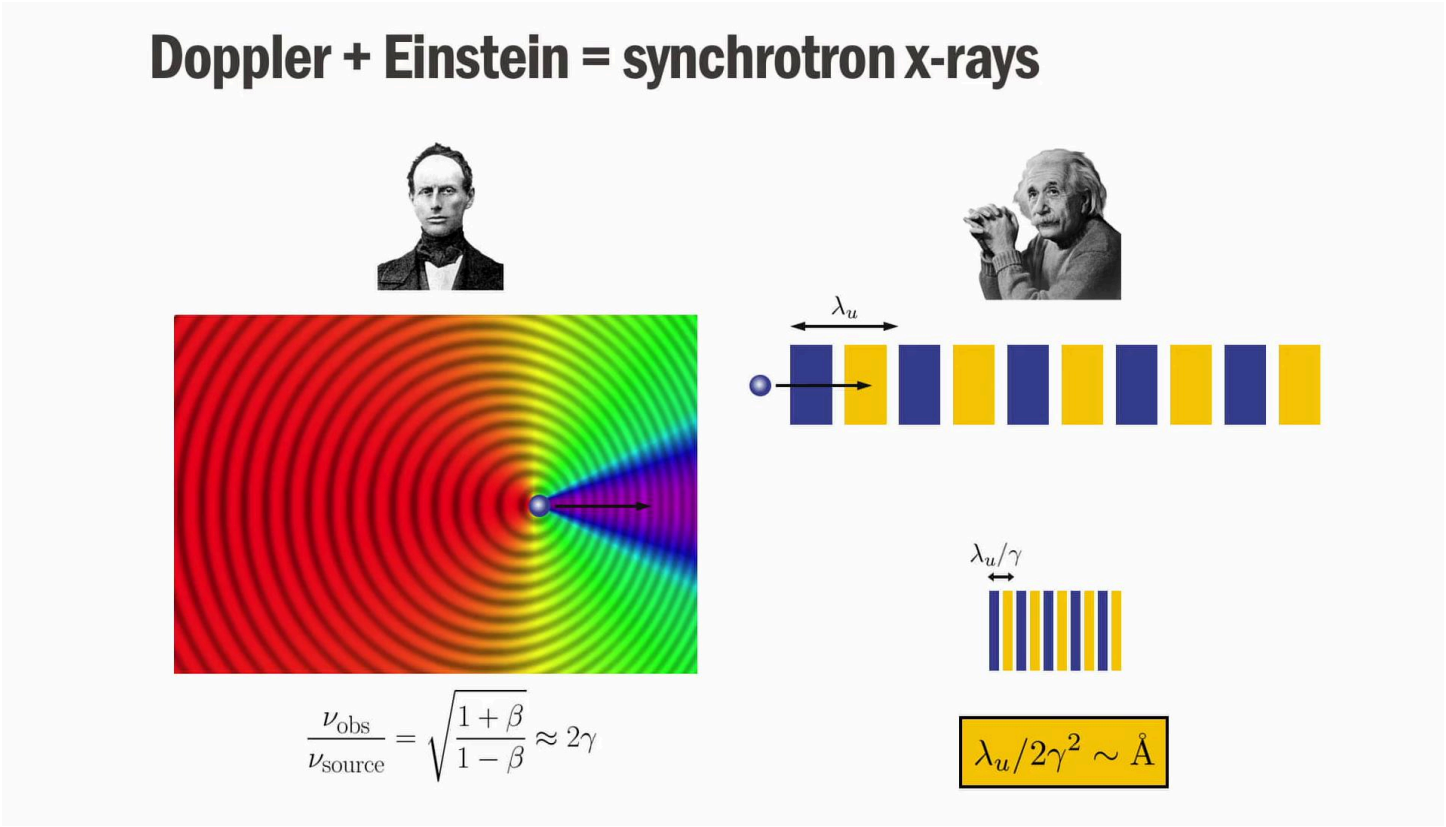


We now turn to the wavelength of the radiation emitted by relativistic electrons, two phenomena combined to shift the radiation from the microwave regime to x-rays.

Notes

Summary

12m 24s




First, Doppler blue shifting increases the frequency by a factor of the square root of 1 plus Beta upon 1 minus Beta, which it is easy to demonstrate is accurately approximated by 2 Gamma. Secondly, so-called relativistic Lorentz contraction reduces the length scale in the direction of propagation by a factor of 1 upon Gamma. Together, these therefore reduce the characteristic length scales by a factor of 1 upon 2 Gammo squared. But what is a characteristic length scale? Well, in the case of undulators in the [inaudible 00:13:17] of synchotron rings. This is the undulator periodicity Lamda U. This is the repeat distance of a north- south and south-north magnet dipole pair in the long array of such alternating magnet pairs, which is typically measured in a few centimetres. If the electrons travelling along and undulator were to do so at modest non relativistic velocities, the slalom path they would execute as a consequence of the alternating north-south, south-north magnetic fields of the undulator would result in dipole radiation with the same wavelength. In other words, in the microwave regime. But due to Doppler shifting and Lorentz contraction, this periodicity is reduced to one 50 millionth of this for a gamma of 5000, equating to a wavelength of a few angstroms. This is a source of X-rays.

Notes

Summary

12m 37s

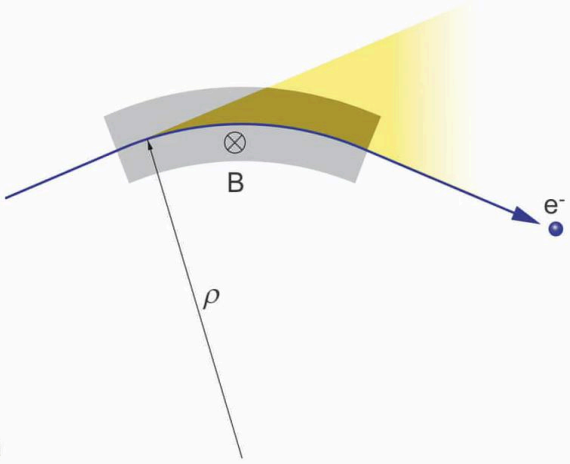
# I got the power

$$\frac{dP}{d\Omega}(\theta = 0) \propto \gamma^6$$

- Assumes a constant acceleration a. Also proportional to $a^2$

$$a = c^2/\rho = \frac{ceB}{\gamma m_e}$$

- e.g. an increase in storage-ring energy by 25% (2.4 GeV to 3.0 GeV) results in an increase in central-cone power density of a factor 3.8

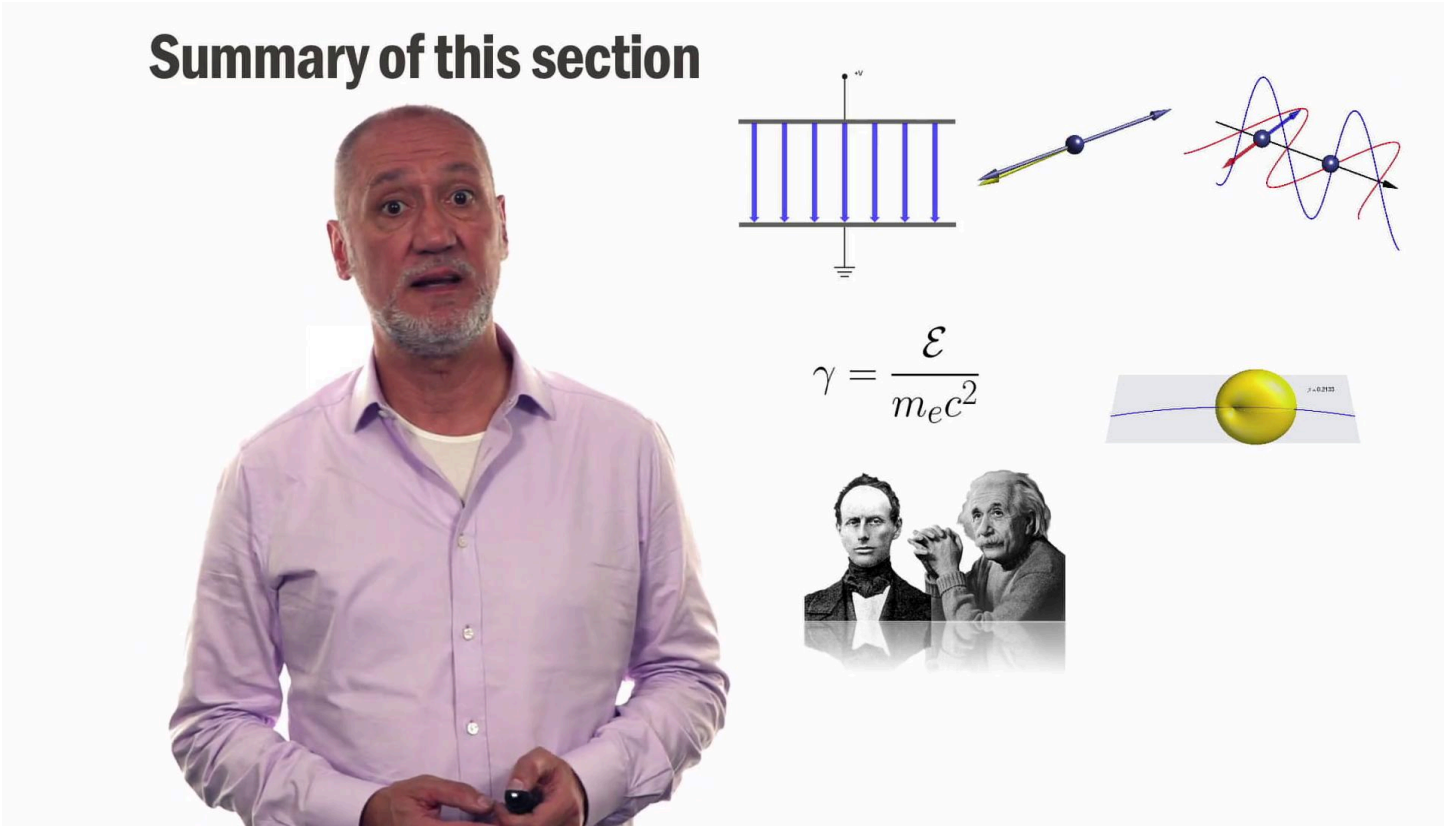



The range of power output per solid angle in the forward direction also increases dramatically with electron energy. It can be shown that for a given centripetal acceleration, this is proportional to the 6th power of Gamma. It's also proportional to the square of the acceleration. But as we will shortly derive for ourselves, the acceleration itself is inversely proportional to Gamma. So keeping this constant, requires a proportional increase in the bending magnetic field strength in order to maintain a constant bending radius Ro. That being the case, a 25% increase in storage ring energy would produce a factor of 3.8 increase in an axis power density.

Notes

Summary

14m 15s



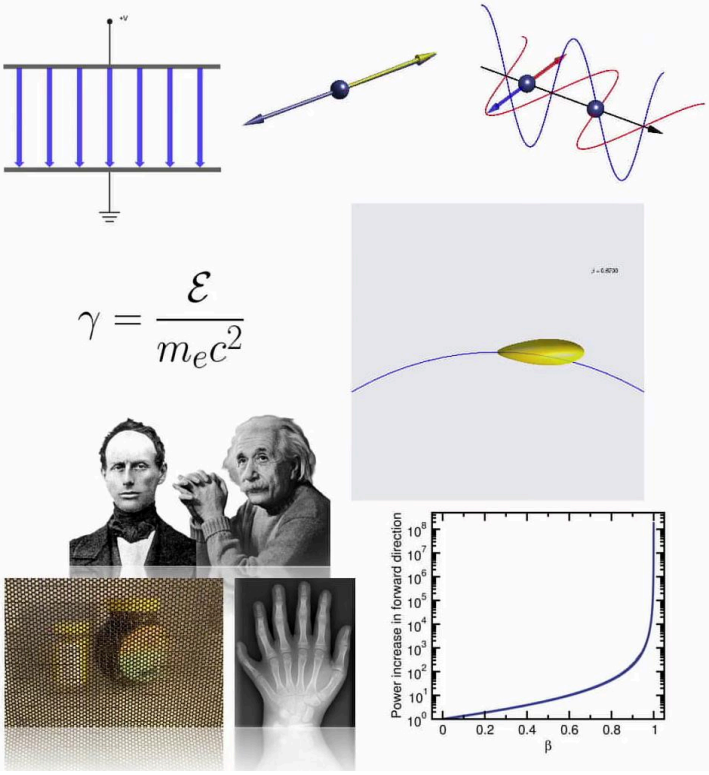

To summarise this section, we began by reviewing electric and magnetic fields and forces also in the context of electromagnetic waves. We then became familiar with the important parameter Gamma in synchrotron physics, expressing the ratio of the electron energy to its rest mass energy. We also saw that at Gamma values much larger than unity, space becomes distorted due to Lorentz contraction, causing, for example, the form of radiation emitted by a centripetally accelerated electron to transform from dipole radiation to the highly forward directed synchrotron radiation at relativistic velocities. The wavelength of this radiation is reduced by several orders of magnitude due to both Doppler, blue shifting and Lorentz contraction, converting what would otherwise be a weak source of microwave radiation into an intense and highly collimated X-ray source.

Notes

Summary

15m 05s

# Summary of this section




We now have the necessary background knowledge to understand the properties and design of the various components of the storage ring. However, before we apply this to take a closer look at the most important elements of the so-called magnet Latisse in the last section of this week, we need in the next section to first quantify the most important properties of synchrotron radiation, namely the spectral flux, the emittance, the brightness and the coherent properties.

Notes

Summary

16m 09s